GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2018

10611

MATHEMATICS

PAPER 1

TIME: 3 hours

MARKS: 150

9 pages and 1 information sheet

MATHEMATICS: Paper 1
1061E
10611E

X05

P.T.O

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(Paper 1)

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which were used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical).
6. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet is included on Page 10 of the question paper.
9. Number the questions correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 30 = 0$ (2)

1.1.2 $3x^2 - 8x = 4$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{5-x} - x = 1$ (5)

1.1.4 $\frac{6x^2 - 3x}{3} \leq 3x^2$ (5)

1.1.5 $2^{x+2} + 7\sqrt{2^x} = 2$ (5)

1.2 Prove that the equation $6x^2 + 2px - 3x - p = 0$ has rational roots for all rational values of p. (4)

[25]

QUESTION 2

2.1 Calculate the number of terms in the following arithmetic sequence:

6; 1; –4; –9; ... ; –239 (3)

2.2 The 3rd term of a geometric series is 18 and the 5th term is 162.
Determine the sum of the first 7 terms, where $r < 0$. (6)

2.3 The following terms form a quadratic sequence:
3; x; 11; 21; 35; ...
Determine the value of x. (3)

2.4 The first term of a geometric sequence is 9. The ratio of the sum of the first eight terms to the sum of the first four terms is 97 : 81.
Determine the first THREE terms of the sequence, if all terms are positive. (6)

2.5 Consider the infinite geometric series:
$2(p-5) + 2(p-5)^2 + 2(p-5)^3 + \ldots$

2.5.1 For which value(s) of p is the series convergent? (3)

2.5.2 If $p = 4\frac{1}{2}$, calculate S_∞. (3)

[24]

QUESTION 3

3.1 Lungile bought a car for R134 000. It depreciates on a reducing balance method at a rate of 6,8% per annum.
After how many years will its value be R100 000? (4)

3.2 A bank granted Clive a loan of R150 000 at an interest rate of 15,25% per annum, compounded monthly. Clive will repay the loan in 24 equal monthly payments. Payments will start 3 months after the loan was granted.

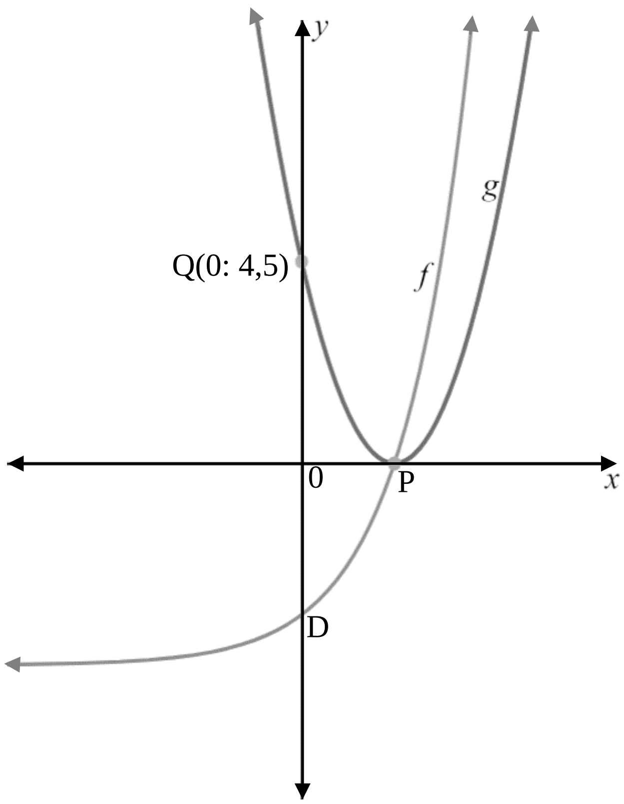
3.2.1 Calculate his monthly payment. (5)

3.2.2 Calculate the balance outstanding immediately after Clive makes his 18^{th} payment. (4)

[13]

QUESTION 4

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below.
Point Q (0 ; 4,5) and point D are the y – intercepts of graphs g and f respectively.
The graphs intersect at point P, which is the turning point of graph g and the common x – intercept of f and g.



4.1 Write down the equation of the asymptote of graph f. (1)

4.2 Determine the coordinates of point P and point D. (4)

4.3 Determine the equation of h if $h(x) = f(2x) + 8$. (2)

4.4 Determine the equation of h^{-1} in the form $y = ...$ (2)

4.5 Write down the range of h^{-1}. (1)

4.6 Determine the equation of g. (3)

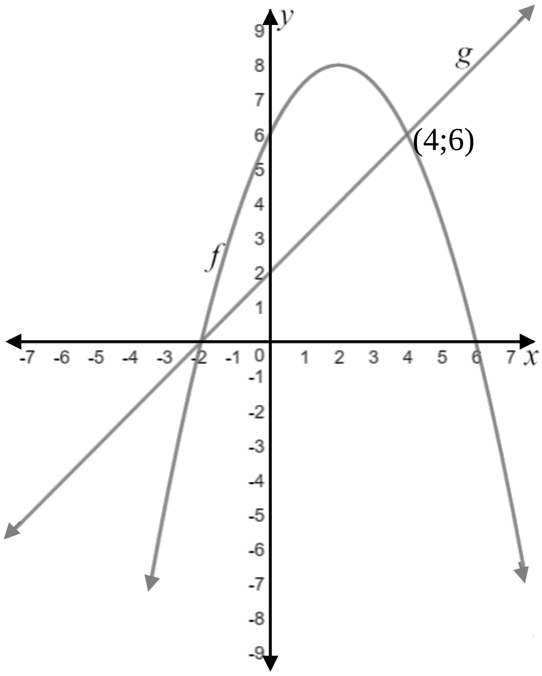
4.7 Calculate: $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ (3)

4.8 Describe the transformation that should be applied to graph g so that the new graph obtained will have non-real roots? (1)

[17]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}x^2 + 2x + 6$ and $g(x) = x + 2$ are sketched below. The graphs intersect at (–2; 0) and (4; 6).



Use the graphs to determine the values of *x* for which:

5.1 $f(x) = g(x)$ (2)

5.2 $\dfrac{f(x)}{g(x)} \geq 0$ (2)

5.3 $f'(x) \cdot g(x) \geq 0$ (2)

[6]

QUESTION 6

Given: $f(x) = \frac{1}{4}x^2$

6.1 Write down the equation of g if g is the reflection of f about the y-axis. (1)

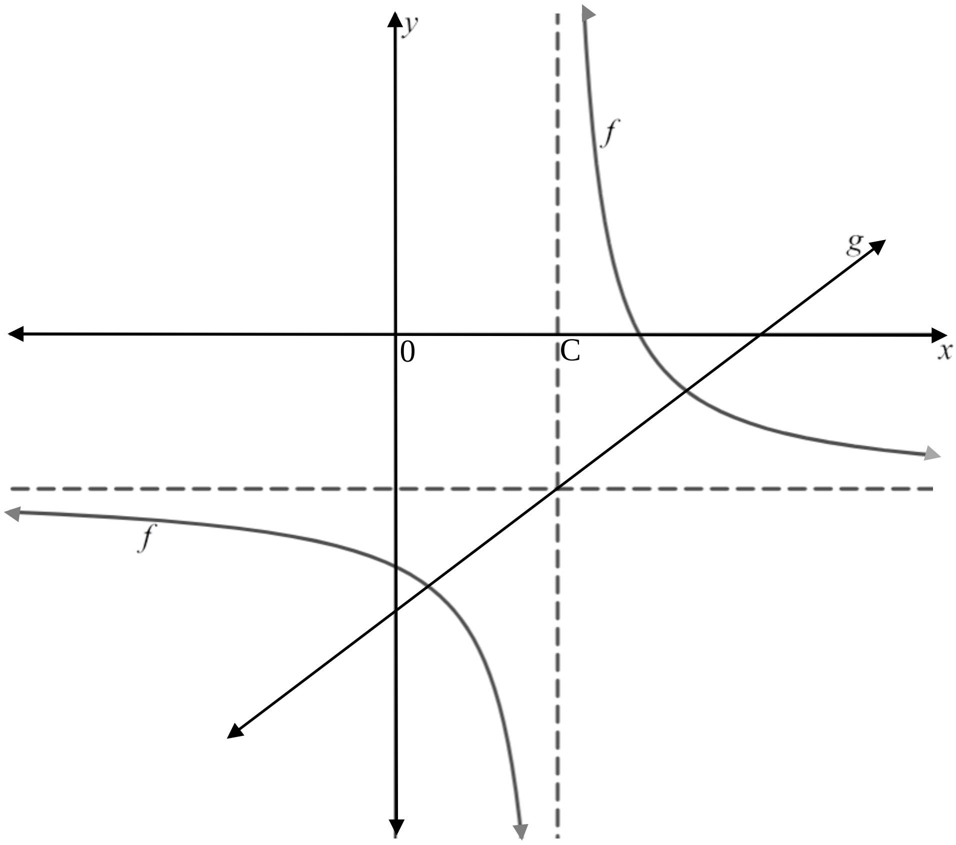
6.2 Write down the equation of h if f is translated TWO units down to obtain h. (1)

6.3 Write down the range of h. (1)

[3]

QUESTION 7

The graphs of $f(x)=\dfrac{3}{x-2}-3$ and g, an axis of symmetry of f, are sketched below. The vertical asymptote cuts the x-axis at C.



7.1 Write down the equation of the vertical asymptote of f. (1)

7.2 Describe how the graph of $h(x)=\dfrac{3}{x}$ was transformed to obtain f. (2)

7.3 Write down the domain of $f(x-1)$. (1)

7.4 Determine the equation of the line, parallel to g (an axis of symmetry of f) passing through point C. (3)
[7]

QUESTION 8

Given: $f(x)=1-3x^2$

8.1 Determine $f'(x)$ from FIRST principles. (5)

8.2 Hence, calculate the gradient of a tangent to f at $x=2$. (2)
[7]

QUESTION 9

Determine the following:

9.1 $\frac{d}{dt}\left[(t-2)(t+3)\right]$ (3)

9.2 $D_x\left[\frac{5x^3-4}{x}\right]$ (3)

[6]

QUESTION 10

The gradient of a tangent to the curve $f(x) = ax^3 + bx^2$ at point C (1 ; 7) is 17.

10.1 Calculate the values of a and b. (6)

10.2 If it is given that $a = 3$ and $b = 4$, determine the coordinate of one other point on the curve where the gradient of the curve is also equal to 17. (6)

10.3 Sketch the graph of $f(x) = 3x^3 + 4x^2$, indicating all intercepts with the axes as well as the turning points. (4)

10.4 Calculate the values of x for which $f(x) = 3x^3 + 4x^2$ is concave up. (3)

[19]

QUESTION 11

The path travelled by a meteor can be tracked using the formula:
$s(t) = 6000 - 600t - 0{,}2t^3 + 2\times10^{-3}t^5$, where $s(t)$ is the distance (in meters) that the meteor is from the earth, t seconds after being detected.

11.1 Determine the velocity at which the meteor approaches the earth when FIRST detected. (3)

11.2 Show that the meteor will collide with the earth at $t = 10s$. (2)

11.3 Determine the acceleration (rate of change of velocity) of the meteor at $t = 5s$. (3)

[8]

QUESTION 12

Events A, B and C occur as follows where A and B are independent events.

- $P(A) = 0{,}38$
- $P(B) = 0{,}42$
- $P(A \cap B) = 0{,}1596$
- $P(C) = 0{,}28$
- There are 456 people in event A.

12.1 Are A and B mutually exclusive events? Motivate your answer. (2)

12.2 By using an appropriate formula, show that the value of $P(A \cup B) = 0{,}64$. (2)

12.3 Calculate the number of people in the sample space. (2)

12.4 Determine $n(C')$. (2)

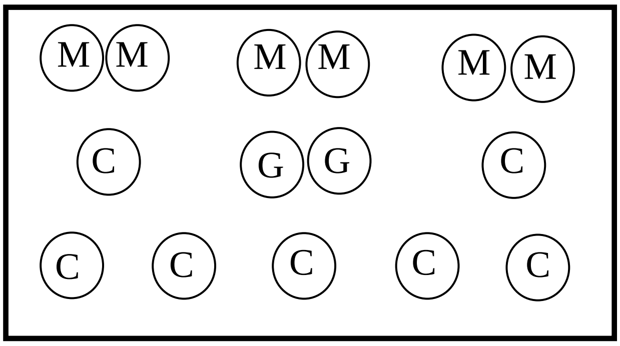
[8]

QUESTION 13

13.1 The letters in the word JOHAN are arranged in any order WITHOUT repetition. What is the probability that the word JOHAN will start with the letter J and end with the letter A? (3)

13.2 The Lauwrens family takes family photos. The photographer arranges three married couples, seven children and two grandparents as follows:

The couples stand husband and wife together at the back, the grandparents in the middle and the children in the other positions as shown in the diagram below.



M	Married Couples
G	Grandparents
C	Children

How many different ways can the Lauwrens family be arranged for the photo? (4)

[7]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x ; y) \to (x\cos\theta - y\sin\theta \ ; \ y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$